Mechanics

PART 1

Physics, the most fundamental physical science, is concerned with the basic principles of the Universe. It is the foundation upon which the other sciences—astronomy, biology, chemistry, and geology—are based. The beauty of physics lies in the simplicity of the fundamental physical theories and in the manner in which just a small number of fundamental concepts, equations, and assumptions can alter and expand our view of the world around us.

The study of physics can be divided into six main areas:

1. *classical mechanics,* which is concerned with the motion of objects that are large relative to atoms and move at speeds much slower than the speed of light;
2. *relativity,* which is a theory describing objects moving at any speed, even speeds approaching the speed of light;
3. *thermodynamics,* which deals with heat, work, temperature, and the statistical behavior of systems with large numbers of particles;
4. *electromagnetism,* which is concerned with electricity, magnetism, and electromagnetic fields;
5. *optics,* which is the study of the behavior of light and its interaction with materials;
6. *quantum mechanics,* a collection of theories connecting the behavior of matter at the submicroscopic level to macroscopic observations.

The disciplines of mechanics and electromagnetism are basic to all other branches of classical physics (developed before 1900) and modern physics (c. 1900–present). The first part of this textbook deals with classical mechanics, sometimes referred to as *Newtonian mechanics* or simply *mechanics.* This is an appropriate place to begin an introductory text because many of the basic principles used to understand mechanical systems can later be used to describe such natural phenomena as waves and the transfer of energy by heat. Furthermore, the laws of conservation of energy and momentum introduced in mechanics retain their importance in the fundamental theories of other areas of physics.

Today, classical mechanics is of vital importance to students from all disciplines. It is highly successful in describing the motions of different objects, such as planets, rockets, and baseballs. In the first part of the text, we shall describe the laws of classical mechanics and examine a wide range of phenomena that can be understood with these fundamental ideas.

◀ *Liftoff of the space shuttle* Columbia. *The tragic accident of February 1, 2003 that took the lives of all seven astronauts aboard happened just before Volume 1 of this book went to press. The launch and operation of a space shuttle involves many fundamental principles of classical mechanics, thermodynamics, and electromagnetism. We study the principles of classical mechanics in Part 1 of this text, and apply these principles to rocket propulsion in Chapter 9. (NASA)*

Chapter 1

Physics and Measurement

CHAPTER OUTLINE

▲ *The workings of a mechanical clock. Complicated timepieces have been built for centuries in an effort to measure time accurately. Time is one of the basic quantities that we use in studying the motion of objects. (elektraVision/Index Stock Imagery)*

Like all other sciences, physics is based on experimental observations and quantitative measurements. The main objective of physics is to find the limited number of fundamental laws that govern natural phenomena and to use them to develop theories that can predict the results of future experiments. The fundamental laws used in developing theories are expressed in the language of mathematics, the tool that provides a bridge between theory and experiment.

When a discrepancy between theory and experiment arises, new theories must be formulated to remove the discrepancy. Many times a theory is satisfactory only under limited conditions; a more general theory might be satisfactory without such limitations. For example, the laws of motion discovered by Isaac Newton (1642–1727) in the 17th century accurately describe the motion of objects moving at normal speeds but do not apply to objects moving at speeds comparable with the speed of light. In contrast, the special theory of relativity developed by Albert Einstein (1879–1955) in the early 1900s gives the same results as Newton's laws at low speeds but also correctly describes motion at speeds approaching the speed of light. Hence, Einstein's special theory of relativity is a more general theory of motion.

Classical physics includes the theories, concepts, laws, and experiments in classical mechanics, thermodynamics, optics, and electromagnetism developed before 1900. Important contributions to classical physics were provided by Newton, who developed classical mechanics as a systematic theory and was one of the originators of calculus as a mathematical tool. Major developments in mechanics continued in the 18th century, but the fields of thermodynamics and electricity and magnetism were not developed until the latter part of the 19th century, principally because before that time the apparatus for controlled experiments was either too crude or unavailable.

A major revolution in physics, usually referred to as *modern physics,* began near the end of the 19th century. Modern physics developed mainly because of the discovery that many physical phenomena could not be explained by classical physics. The two most important developments in this modern era were the theories of relativity and quantum mechanics. Einstein's theory of relativity not only correctly described the motion of objects moving at speeds comparable to the speed of light but also completely revolutionized the traditional concepts of space, time, and energy. The theory of relativity also shows that the speed of light is the upper limit of the speed of an object and that mass and energy are related. Quantum mechanics was formulated by a number of distinguished scientists to provide descriptions of physical phenomena at the atomic level.

Scientists continually work at improving our understanding of fundamental laws, and new discoveries are made every day. In many research areas there is a great deal of overlap among physics, chemistry, and biology. Evidence for this overlap is seen in the names of some subspecialties in science—biophysics, biochemistry, chemical physics, biotechnology, and so on. Numerous technological advances in recent times are the result of the efforts of many scientists, engineers, and technicians. Some of the most notable developments in the latter half of the 20th century were (1) unmanned planetary explorations and manned moon landings, (2) microcircuitry and high-speed computers, (3) sophisticated imaging techniques used in scientific research and medicine, and

(4) several remarkable results in genetic engineering. The impacts of such developments and discoveries on our society have indeed been great, and it is very likely that future discoveries and developments will be exciting, challenging, and of great benefit to humanity.

1.1 Standards of Length, Mass, and Time

The laws of physics are expressed as mathematical relationships among physical quantities that we will introduce and discuss throughout the book. Most of these quantities are *derived quantities,* in that they can be expressed as combinations of a small number of *basic quantities.* In mechanics, the three basic quantities are length, mass, and time. All other quantities in mechanics can be expressed in terms of these three.

If we are to report the results of a measurement to someone who wishes to reproduce this measurement, a *standard* must be defined. It would be meaningless if a visitor from another planet were to talk to us about a length of 8 "glitches" if we do not know the meaning of the unit glitch. On the other hand, if someone familiar with our system of measurement reports that a wall is 2 meters high and our unit of length is defined to be 1 meter, we know that the height of the wall is twice our basic length unit. Likewise, if we are told that a person has a mass of 75 kilograms and our unit of mass is defined to be 1 kilogram, then that person is 75 times as massive as our basic unit.[1] Whatever is chosen as a standard must be readily accessible and possess some property that can be measured reliably. Measurements taken by different people in different places must yield the same result.

In 1960, an international committee established a set of standards for the fundamental quantities of science. It is called the **SI** (Système International), and its units of length, mass, and time are the *meter, kilogram,* and *second,* respectively. Other SI standards established by the committee are those for temperature (the *kelvin*), electric current (the *ampere*), luminous intensity (the *candela*), and the amount of substance (the *mole*).

Length

In A.D. 1120 the king of England decreed that the standard of length in his country would be named the *yard* and would be precisely equal to the distance from the tip of his nose to the end of his outstretched arm. Similarly, the original standard for the foot adopted by the French was the length of the royal foot of King Louis XIV. This standard prevailed until 1799, when the legal standard of length in France became the *meter,* defined as one ten-millionth the distance from the equator to the North Pole along one particular longitudinal line that passes through Paris.

Many other systems for measuring length have been developed over the years, but the advantages of the French system have caused it to prevail in almost all countries and in scientific circles everywhere. As recently as 1960, the length of the meter was defined as the distance between two lines on a specific platinum–iridium bar stored under controlled conditions in France. This standard was abandoned for several reasons, a principal one being that the limited accuracy with which the separation between the lines on the bar can be determined does not meet the current requirements of science and technology. In the 1960s and 1970s, the meter was defined as 1 650 763.73 wavelengths of orange-red light emitted from a krypton-86 lamp. However, in October 1983, **the meter (m) was redefined as the distance traveled by light in vacuum during a time of 1/299 792 458 second.** In effect, this

[1] The need for assigning numerical values to various measured physical quantities was expressed by Lord Kelvin (William Thomson) as follows: "I often say that when you can measure what you are speaking about, and express it in numbers, you should know something about it, but when you cannot express it in numbers, your knowledge is of a meager and unsatisfactory kind. It may be the beginning of knowledge but you have scarcely in your thoughts advanced to the state of science."

Table 1.1

Approximate Values of Some Measured Lengths

	Length (m)
Distance from the Earth to the most remote known quasar	1.4×10^{26}
Distance from the Earth to the most remote normal galaxies	9×10^{25}
Distance from the Earth to the nearest large galaxy (M 31, the Andromeda galaxy)	2×10^{22}
Distance from the Sun to the nearest star (Proxima Centauri)	4×10^{16}
One lightyear	9.46×10^{15}
Mean orbit radius of the Earth about the Sun	1.50×10^{11}
Mean distance from the Earth to the Moon	3.84×10^{8}
Distance from the equator to the North Pole	1.00×10^{7}
Mean radius of the Earth	6.37×10^{6}
Typical altitude (above the surface) of a satellite orbiting the Earth	2×10^{5}
Length of a football field	9.1×10^{1}
Length of a housefly	5×10^{-3}
Size of smallest dust particles	$\sim 10^{-4}$
Size of cells of most living organisms	$\sim 10^{-5}$
Diameter of a hydrogen atom	$\sim 10^{-10}$
Diameter of an atomic nucleus	$\sim 10^{-14}$
Diameter of a proton	$\sim 10^{-15}$

latest definition establishes that the speed of light in vacuum is precisely 299 792 458 meters per second.

Table 1.1 lists approximate values of some measured lengths. You should study this table as well as the next two tables and begin to generate an intuition for what is meant by a length of 20 centimeters, for example, or a mass of 100 kilograms or a time interval of 3.2×10^7 seconds.

Mass

The SI unit of mass, **the kilogram (kg), is defined as the mass of a specific platinum–iridium alloy cylinder kept at the International Bureau of Weights and Measures at Sèvres, France.** This mass standard was established in 1887 and has not been changed since that time because platinum–iridium is an unusually stable alloy. A duplicate of the Sèvres cylinder is kept at the National Institute of Standards and Technology (NIST) in Gaithersburg, Maryland (Fig. 1.1a).

Table 1.2 lists approximate values of the masses of various objects.

Time

Before 1960, the standard of time was defined in terms of the *mean solar day* for the year 1900. (A solar day is the time interval between successive appearances of the Sun at the highest point it reaches in the sky each day.) The *second* was defined as $\left(\frac{1}{60}\right)\left(\frac{1}{60}\right)\left(\frac{1}{24}\right)$ of a mean solar day. The rotation of the Earth is now known to vary slightly with time, however, and therefore this motion is not a good one to use for defining a time standard.

In 1967, the second was redefined to take advantage of the high precision attainable in a device known as an *atomic clock* (Fig. 1.1b), which uses the characteristic frequency of the cesium-133 atom as the "reference clock." **The second (s) is now defined as 9 192 631 770 times the period of vibration of radiation from the cesium atom.**[2]

[2] *Period* is defined as the time interval needed for one complete vibration.

PITFALL PREVENTION

1.1 No Commas in Numbers with Many Digits

We will use the standard scientific notation for numbers with more than three digits, in which groups of three digits are separated by spaces rather than commas. Thus, 10 000 is the same as the common American notation of 10,000. Similarly, $\pi = 3.14159265$ is written as 3.141 592 65.

Table 1.2

Masses of Various Objects (Approximate Values)

	Mass (kg)
Observable Universe	$\sim 10^{52}$
Milky Way galaxy	$\sim 10^{42}$
Sun	1.99×10^{30}
Earth	5.98×10^{24}
Moon	7.36×10^{22}
Shark	$\sim 10^{3}$
Human	$\sim 10^{2}$
Frog	$\sim 10^{-1}$
Mosquito	$\sim 10^{-5}$
Bacterium	$\sim 1 \times 10^{-15}$
Hydrogen atom	1.67×10^{-27}
Electron	9.11×10^{-31}

PITFALL PREVENTION

1.2 Reasonable Values

Generating intuition about typical values of quantities is important because when solving problems you must think about your end result and determine if it seems reasonable. If you are calculating the mass of a housefly and arrive at a value of 100 kg, this is *unreasonable*—there is an error somewhere.

(Courtesy of National Institute of Standards and Technology, U.S. Department of Commerce)

(a) (b)

Figure 1.1 (a) The National Standard Kilogram No. 20, an accurate copy of the International Standard Kilogram kept at Sèvres, France, is housed under a double bell jar in a vault at the National Institute of Standards and Technology. (b) The nation's primary time standard is a cesium fountain atomic clock developed at the National Institute of Standards and Technology laboratories in Boulder, Colorado. The clock will neither gain nor lose a second in 20 million years.

To keep these atomic clocks—and therefore all common clocks and watches that are set to them—synchronized, it has sometimes been necessary to add leap seconds to our clocks.

Since Einstein's discovery of the linkage between space and time, precise measurement of time intervals requires that we know both the state of motion of the clock used to measure the interval and, in some cases, the location of the clock as well. Otherwise, for example, global positioning system satellites might be unable to pinpoint your location with sufficient accuracy, should you need to be rescued.

Approximate values of time intervals are presented in Table 1.3.

Table 1.3

Approximate Values of Some Time Intervals

	Time Interval (s)
Age of the Universe	5×10^{17}
Age of the Earth	1.3×10^{17}
Average age of a college student	6.3×10^{8}
One year	3.2×10^{7}
One day (time interval for one revolution of the Earth about its axis)	8.6×10^{4}
One class period	3.0×10^{3}
Time interval between normal heartbeats	8×10^{-1}
Period of audible sound waves	$\sim 10^{-3}$
Period of typical radio waves	$\sim 10^{-6}$
Period of vibration of an atom in a solid	$\sim 10^{-13}$
Period of visible light waves	$\sim 10^{-15}$
Duration of a nuclear collision	$\sim 10^{-22}$
Time interval for light to cross a proton	$\sim 10^{-24}$

Table 1.4

Prefixes for Powers of Ten

Power	Prefix	Abbreviation
10^{-24}	yocto	y
10^{-21}	zepto	z
10^{-18}	atto	a
10^{-15}	femto	f
10^{-12}	pico	p
10^{-9}	nano	n
10^{-6}	micro	μ
10^{-3}	milli	m
10^{-2}	centi	c
10^{-1}	deci	d
10^{3}	kilo	k
10^{6}	mega	M
10^{9}	giga	G
10^{12}	tera	T
10^{15}	peta	P
10^{18}	exa	E
10^{21}	zetta	Z
10^{24}	yotta	Y

In addition to SI, another system of units, the *U.S. customary system,* is still used in the United States despite acceptance of SI by the rest of the world. In this system, the units of length, mass, and time are the foot (ft), slug, and second, respectively. In this text we shall use SI units because they are almost universally accepted in science and industry. We shall make some limited use of U.S. customary units in the study of classical mechanics.

In addition to the basic SI units of meter, kilogram, and second, we can also use other units, such as millimeters and nanoseconds, where the prefixes *milli-* and *nano-* denote multipliers of the basic units based on various powers of ten. Prefixes for the various powers of ten and their abbreviations are listed in Table 1.4. For example, 10^{-3} m is equivalent to 1 millimeter (mm), and 10^3 m corresponds to 1 kilometer (km). Likewise, 1 kilogram (kg) is 10^3 grams (g), and 1 megavolt (MV) is 10^6 volts (V).

1.2 Matter and Model Building

If physicists cannot interact with some phenomenon directly, they often imagine a **model** for a physical system that is related to the phenomenon. In this context, a model is a system of physical components, such as electrons and protons in an atom. Once we have identified the physical components, we make predictions about the behavior of the system, based on the interactions among the components of the system and/or the interaction between the system and the environment outside the system.

As an example, consider the behavior of *matter.* A 1-kg cube of solid gold, such as that at the left of Figure 1.2, has a length of 3.73 cm on a side. Is this cube nothing but wall-to-wall gold, with no empty space? If the cube is cut in half, the two pieces still retain their chemical identity as solid gold. But what if the pieces are cut again and again, indefinitely? Will the smaller and smaller pieces always be gold? Questions such as these can be traced back to early Greek philosophers. Two of them—Leucippus and his student Democritus—could not accept the idea that such cuttings could go on forever. They speculated that the process ultimately must end when it produces a particle

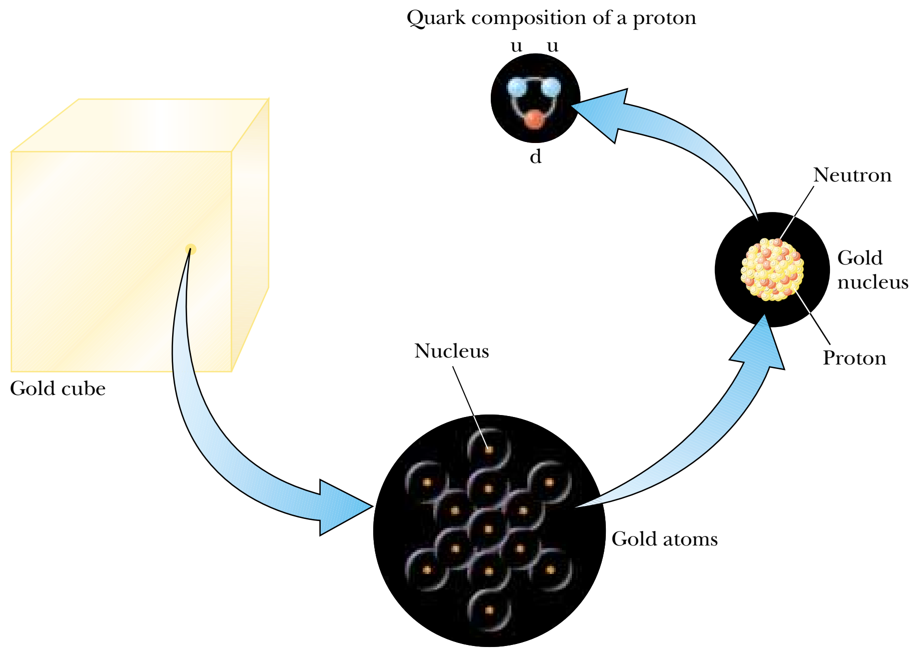

Figure 1.2 Levels of organization in matter. Ordinary matter consists of atoms, and at the center of each atom is a compact nucleus consisting of protons and neutrons. Protons and neutrons are composed of quarks. The quark composition of a proton is shown.

that can no longer be cut. In Greek, *atomos* means "not sliceable." From this comes our English word *atom.*

Let us review briefly a number of historical models of the structure of matter. The Greek model of the structure of matter was that all ordinary matter consists of atoms, as suggested to the lower right of the cube in Figure 1.2. Beyond that, no additional structure was specified in the model—atoms acted as small particles that interacted with each other, but internal structure of the atom was not a part of the model.

In 1897, J. J. Thomson identified the electron as a charged particle and as a constituent of the atom. This led to the first model of the atom that contained internal structure. We shall discuss this model in Chapter 42.

Following the discovery of the nucleus in 1911, a model was developed in which each atom is made up of electrons surrounding a central nucleus. A nucleus is shown in Figure 1.2. This model leads, however, to a new question—does the nucleus have structure? That is, is the nucleus a single particle or a collection of particles? The exact composition of the nucleus is not known completely even today, but by the early 1930s a model evolved that helped us understand how the nucleus behaves. Specifically, scientists determined that occupying the nucleus are two basic entities, protons and neutrons. The proton carries a positive electric charge, and a specific chemical element is identified by the number of protons in its nucleus. This number is called the **atomic number** of the element. For instance, the nucleus of a hydrogen atom contains one proton (and so the atomic number of hydrogen is 1), the nucleus of a helium atom contains two protons (atomic number 2), and the nucleus of a uranium atom contains 92 protons (atomic number 92). In addition to atomic number, there is a second number characterizing atoms—**mass number,** defined as the number of protons plus neutrons in a nucleus. The atomic number of an element never varies (i.e., the number of protons does not vary) but the mass number can vary (i.e., the number of neutrons varies).

The existence of neutrons was verified conclusively in 1932. A neutron has no charge and a mass that is about equal to that of a proton. One of its primary purposes is to act as a "glue" that holds the nucleus together. If neutrons were not present in the nucleus, the repulsive force between the positively charged particles would cause the nucleus to come apart.

But is this where the process of breaking down stops? Protons, neutrons, and a host of other exotic particles are now known to be composed of six different varieties of particles called **quarks,** which have been given the names of *up, down, strange, charmed, bottom,* and *top*. The up, charmed, and top quarks have electric charges of $+\frac{2}{3}$ that of the proton, whereas the down, strange, and bottom quarks have charges of $-\frac{1}{3}$ that of the proton. The proton consists of two up quarks and one down quark, as shown at the top in Figure 1.2. You can easily show that this structure predicts the correct charge for the proton. Likewise, the neutron consists of two down quarks and one up quark, giving a net charge of zero.

This process of building models is one that you should develop as you study physics. You will be challenged with many mathematical problems to solve in this study. One of the most important techniques is to build a model for the problem—identify a system of physical components for the problem, and make predictions of the behavior of the system based on the interactions among the components of the system and/or the interaction between the system and its surrounding environment.

1.3 Density and Atomic Mass

In Section 1.1, we explored three basic quantities in mechanics. Let us look now at an example of a derived quantity—**density.** The density ρ (Greek letter rho) of any substance is defined as its *mass per unit volume:*

A table of the letters in the Greek alphabet is provided on the back endsheet of the textbook.

$$\rho \equiv \frac{m}{V} \tag{1.1}$$

For example, aluminum has a density of $2.70\ \mathrm{g/cm^3}$, and lead has a density of $11.3\ \mathrm{g/cm^3}$. Therefore, a piece of aluminum of volume $10.0\ \mathrm{cm^3}$ has a mass of 27.0 g, whereas an equivalent volume of lead has a mass of 113 g. A list of densities for various substances is given in Table 1.5.

The numbers of protons and neutrons in the nucleus of an atom of an element are related to the **atomic mass** of the element, which is defined as the mass of a single atom of the element measured in **atomic mass units** (u) where $1\ \mathrm{u} = 1.660\,538\,7 \times 10^{-27}$ kg.

Table 1.5

Densities of Various Substances

Substance	**Density ρ ($10^3\ \mathrm{kg/m^3}$)**
Platinum	21.45
Gold	19.3
Uranium	18.7
Lead	11.3
Copper	8.92
Iron	7.86
Aluminum	2.70
Magnesium	1.75
Water	1.00
Air at atmospheric pressure	0.0012

The atomic mass of lead is 207 u and that of aluminum is 27.0 u. However, the ratio of atomic masses, 207 u/27.0 u = 7.67, does not correspond to the ratio of densities, $(11.3 \times 10^3 \text{ kg/m}^3)/(2.70 \times 10^3 \text{ kg/m}^3) = 4.19$. This discrepancy is due to the difference in atomic spacings and atomic arrangements in the crystal structures of the two elements.

Quick Quiz 1.1 In a machine shop, two cams are produced, one of aluminum and one of iron. Both cams have the same mass. Which cam is larger? (a) the aluminum cam (b) the iron cam (c) Both cams have the same size.

Example 1.1 How Many Atoms in the Cube?

A solid cube of aluminum (density 2.70 g/cm^3) has a volume of 0.200 cm^3. It is known that 27.0 g of aluminum contains 6.02×10^{23} atoms. How many aluminum atoms are contained in the cube?

Solution Because density equals mass per unit volume, the mass of the cube is

$$m = \rho V = (2.70 \text{ g}/\cancel{\text{cm}^3})(0.200 \ \cancel{\text{cm}^3}) = 0.540 \text{ g}$$

To solve this problem, we will set up a ratio based on the fact that the mass of a sample of material is proportional to the number of atoms contained in the sample. This technique of solving by ratios is very powerful and should be studied and understood so that it can be applied in future problem solving. Let us express our proportionality as $m = kN$, where m is the mass of the sample, N is the number of atoms in the sample, and k is an unknown proportionality constant. We write this relationship twice, once for the actual sample of aluminum in the problem and once for a 27.0-g sample, and then we divide the first equation by the second:

$$\begin{aligned} m_{\text{sample}} &= kN_{\text{sample}} \\ m_{27.0 \text{ g}} &= kN_{27.0 \text{ g}} \end{aligned} \quad \longrightarrow \quad \frac{m_{\text{sample}}}{m_{27.0 \text{ g}}} = \frac{N_{\text{sample}}}{N_{27.0 \text{ g}}}$$

Notice that the unknown proportionality constant k cancels, so we do not need to know its value. We now substitute the values:

$$\frac{0.540 \text{ g}}{27.0 \text{ g}} = \frac{N_{\text{sample}}}{6.02 \times 10^{23} \text{ atoms}}$$

$$N_{\text{sample}} = \frac{(0.540 \text{ g})(6.02 \times 10^{23} \text{ atoms})}{27.0 \text{ g}}$$

$$= \boxed{1.20 \times 10^{22} \text{ atoms}}$$

PITFALL PREVENTION

1.3 Setting Up Ratios

When using ratios to solve a problem, keep in mind that *ratios come from equations.* If you start from equations known to be correct and can divide one equation by the other as in Example 1.1 to obtain a useful ratio, you will avoid reasoning errors. So write the known equations first!

1.4 Dimensional Analysis

The word *dimension* has a special meaning in physics. It denotes the physical nature of a quantity. Whether a distance is measured in units of feet or meters or fathoms, it is still a distance. We say its dimension is *length.*

The symbols we use in this book to specify the dimensions of length, mass, and time are L, M, and T, respectively.[3] We shall often use brackets [ ] to denote the dimensions of a physical quantity. For example, the symbol we use for speed in this book is v, and in our notation the dimensions of speed are written $[v] = \text{L/T}$. As another example, the dimensions of area A are $[A] = \text{L}^2$. The dimensions and units of area, volume, speed, and acceleration are listed in Table 1.6. The dimensions of other quantities, such as force and energy, will be described as they are introduced in the text.

In many situations, you may have to derive or check a specific equation. A useful and powerful procedure called *dimensional analysis* can be used to assist in the derivation or to check your final expression. Dimensional analysis makes use of the fact that

[3] The *dimensions* of a quantity will be symbolized by a capitalized, non-italic letter, such as L. The *symbol* for the quantity itself will be italicized, such as L for the length of an object, or t for time.

Table 1.6

Units of Area, Volume, Velocity, Speed, and Acceleration

System	Area (L^2)	Volume (L^3)	Speed (L/T)	Acceleration (L/T^2)
SI	m^2	m^3	m/s	m/s^2
U.S. customary	ft^2	ft^3	ft/s	ft/s^2

dimensions can be treated as algebraic quantities. For example, quantities can be added or subtracted only if they have the same dimensions. Furthermore, the terms on both sides of an equation must have the same dimensions. By following these simple rules, you can use dimensional analysis to help determine whether an expression has the correct form. The relationship can be correct only if the dimensions on both sides of the equation are the same.

To illustrate this procedure, suppose you wish to derive an equation for the position x of a car at a time t if the car starts from rest and moves with constant acceleration a. In Chapter 2, we shall find that the correct expression is $x = \frac{1}{2}at^2$. Let us use dimensional analysis to check the validity of this expression. The quantity x on the left side has the dimension of length. For the equation to be dimensionally correct, the quantity on the right side must also have the dimension of length. We can perform a dimensional check by substituting the dimensions for acceleration, L/T^2 (Table 1.6), and time, T, into the equation. That is, the dimensional form of the equation $x = \frac{1}{2}at^2$ is

$$\mathrm{L} = \frac{\mathrm{L}}{\cancel{\mathrm{T}^2}} \cdot \cancel{\mathrm{T}^2} = \mathrm{L}$$

The dimensions of time cancel as shown, leaving the dimension of length on the right-hand side.

A more general procedure using dimensional analysis is to set up an expression of the form

$$x \propto a^n t^m$$

where n and m are exponents that must be determined and the symbol $\propto$ indicates a proportionality. This relationship is correct only if the dimensions of both sides are the same. Because the dimension of the left side is length, the dimension of the right side must also be length. That is,

$$[a^n t^m] = \mathrm{L} = \mathrm{L}^1\mathrm{T}^0$$

Because the dimensions of acceleration are L/T^2 and the dimension of time is T, we have

$$(\mathrm{L}/\mathrm{T}^2)^n\mathrm{T}^m = \mathrm{L}^1\mathrm{T}^0$$

$$(\mathrm{L}^n\mathrm{T}^{m-2n}) = \mathrm{L}^1\mathrm{T}^0$$

The exponents of L and T must be the same on both sides of the equation. From the exponents of L, we see immediately that $n = 1$. From the exponents of T, we see that $m - 2n = 0$, which, once we substitute for n, gives us $m = 2$. Returning to our original expression $x \propto a^n t^m$, we conclude that $x \propto at^2$. This result differs by a factor of $\frac{1}{2}$ from the correct expression, which is $x = \frac{1}{2}at^2$.

Quick Quiz 1.2 True or False: Dimensional analysis can give you the numerical value of constants of proportionality that may appear in an algebraic expression.

PITFALL PREVENTION

1.4 Symbols for Quantities

Some quantities have a small number of symbols that represent them. For example, the symbol for time is almost always t. Others quantities might have various symbols depending on the usage. Length may be described with symbols such as x, y, and z (for position), r (for radius), a, b, and c (for the legs of a right triangle), ℓ (for the length of an object), d (for a distance), h (for a height), etc.

Example 1.2 Analysis of an Equation

Show that the expression $v = at$ is dimensionally correct, where v represents speed, a acceleration, and t an instant of time.

Solution For the speed term, we have from Table 1.6

$$[v] = \frac{\mathrm{L}}{\mathrm{T}}$$

The same table gives us $\mathrm{L/T^2}$ for the dimensions of acceleration, and so the dimensions of at are

$$[at] = \frac{\mathrm{L}}{\cancel{\mathrm{T^2}}}\cancel{\mathrm{T}} = \frac{\mathrm{L}}{\mathrm{T}}$$

Therefore, the expression is dimensionally correct. (If the expression were given as $v = at^2$ it would be dimensionally *incorrect*. Try it and see!)

Example 1.3 Analysis of a Power Law

Suppose we are told that the acceleration a of a particle moving with uniform speed v in a circle of radius r is proportional to some power of r, say r^n, and some power of v, say v^m. Determine the values of n and m and write the simplest form of an equation for the acceleration.

Solution Let us take a to be

$$a = kr^n v^m$$

where k is a dimensionless constant of proportionality. Knowing the dimensions of a, r, and v, we see that the dimensional equation must be

$$\frac{\mathrm{L}}{\mathrm{T^2}} = \mathrm{L}^n\left(\frac{\mathrm{L}}{\mathrm{T}}\right)^m = \frac{\mathrm{L}^{n+m}}{\mathrm{T}^m}$$

This dimensional equation is balanced under the conditions

$$n + m = \boxed{1} \qquad \text{and} \qquad m = \boxed{2}$$

Therefore $n = -1$, and we can write the acceleration expression as

$$a = kr^{-1}v^2 = \boxed{k\frac{v^2}{r}}$$

When we discuss uniform circular motion later, we shall see that $k = 1$ if a consistent set of units is used. The constant k would not equal 1 if, for example, v were in km/h and you wanted a in $\mathrm{m/s^2}$.

1.5 Conversion of Units

Sometimes it is necessary to convert units from one measurement system to another, or to convert within a system, for example, from kilometers to meters. Equalities between SI and U.S. customary units of length are as follows:

$$1 \text{ mile} = 1\,609 \text{ m} = 1.609 \text{ km} \qquad 1 \text{ ft} = 0.304\,8 \text{ m} = 30.48 \text{ cm}$$

$$1 \text{ m} = 39.37 \text{ in.} = 3.281 \text{ ft} \qquad 1 \text{ in.} = 0.025\,4 \text{ m} = 2.54 \text{ cm (exactly)}$$

A more complete list of conversion factors can be found in Appendix A.

Units can be treated as algebraic quantities that can cancel each other. For example, suppose we wish to convert 15.0 in. to centimeters. Because 1 in. is defined as exactly 2.54 cm, we find that

$$15.0 \text{ in.} = (15.0\ \cancel{\text{in.}})\left(\frac{2.54 \text{ cm}}{1\ \cancel{\text{in.}}}\right) = 38.1 \text{ cm}$$

where the ratio in parentheses is equal to 1. Notice that we choose to put the unit of an inch in the denominator and it cancels with the unit in the original quantity. The remaining unit is the centimeter, which is our desired result.

PITFALL PREVENTION

1.5 Always Include Units

When performing calculations, include the units for every quantity and carry the units through the entire calculation. Avoid the temptation to drop the units early and then attach the expected units once you have an answer. By including the units in every step, you can detect errors if the units for the answer turn out to be incorrect.

Quick Quiz 1.3 The distance between two cities is 100 mi. The number of kilometers between the two cities is (a) smaller than 100 (b) larger than 100 (c) equal to 100.

Example 1.4 Is He Speeding?

On an interstate highway in a rural region of Wyoming, a car is traveling at a speed of 38.0 m/s. Is this car exceeding the speed limit of 75.0 mi/h?

Solution We first convert meters to miles:

$$(38.0\ \cancel{\text{m}}/\text{s})\left(\frac{1\ \text{mi}}{1\,609\ \cancel{\text{m}}}\right) = 2.36 \times 10^{-2}\ \text{mi/s}$$

Now we convert seconds to hours:

$$(2.36 \times 10^{-2}\ \text{mi}/\cancel{\text{s}})\left(\frac{60\ \cancel{\text{s}}}{1\ \cancel{\text{min}}}\right)\left(\frac{60\ \cancel{\text{min}}}{1\ \text{h}}\right) = 85.0\ \text{mi/h}$$

Thus, the car is exceeding the speed limit and should slow down.

What If? What if the driver is from outside the U.S. and is familiar with speeds measured in km/h? What is the speed of the car in km/h?

Answer We can convert our final answer to the appropriate units:

$$(85.0\ \cancel{\text{mi}}/\text{h})\left(\frac{1.609\ \text{km}}{1\ \cancel{\text{mi}}}\right) = 137\ \text{km/h}$$

Figure 1.3 shows the speedometer of an automobile, with speeds in both mi/h and km/h. Can you check the conversion we just performed using this photograph?

Phil Boorman/Getty Images

Figure 1.3 The speedometer of a vehicle that shows speeds in both miles per hour and kilometers per hour.

1.6 Estimates and Order-of-Magnitude Calculations

It is often useful to compute an approximate answer to a given physical problem even when little information is available. This answer can then be used to determine whether or not a more precise calculation is necessary. Such an approximation is usually based on certain assumptions, which must be modified if greater precision is needed. We will sometimes refer to an *order of magnitude* of a certain quantity as the power of ten of the number that describes that quantity. Usually, when an order-of-magnitude calculation is made, the results are reliable to within about a factor of 10. If a quantity increases in value by three orders of magnitude, this means that its value increases by a factor of about $10^3 = 1\,000$. We use the symbol $\sim$ for "is on the order of." Thus,

$$0.008\,6 \sim 10^{-2} \qquad 0.002\,1 \sim 10^{-3} \qquad 720 \sim 10^{3}$$

The spirit of order-of-magnitude calculations, sometimes referred to as "guesstimates" or "ball-park figures," is given in the following quotation: "Make an estimate before every calculation, try a simple physical argument . . . before every derivation, guess the answer to every puzzle."[4] Inaccuracies caused by guessing too low for one number are often canceled out by other guesses that are too high. You will find that with practice your guesstimates become better and better. Estimation problems can be fun to work as you freely drop digits, venture reasonable approximations for

[4] E. Taylor and J. A. Wheeler, *Spacetime Physics: Introduction to Special Relativity,* 2nd ed., San Francisco, W. H. Freeman & Company, Publishers, 1992, p. 20.

unknown numbers, make simplifying assumptions, and turn the question around into something you can answer in your head or with minimal mathematical manipulation on paper. Because of the simplicity of these types of calculations, they can be performed on a *small* piece of paper, so these estimates are often called "back-of-the-envelope calculations."

Example 1.5 Breaths in a Lifetime

Estimate the number of breaths taken during an average life span.

Solution We start by guessing that the typical life span is about 70 years. The only other estimate we must make in this example is the average number of breaths that a person takes in 1 min. This number varies, depending on whether the person is exercising, sleeping, angry, serene, and so forth. To the nearest order of magnitude, we shall choose 10 breaths per minute as our estimate of the average. (This is certainly closer to the true value than 1 breath per minute or 100 breaths per minute.) The number of minutes in a year is approximately

$$1 \text{ yr}\left(\frac{400 \text{ days}}{1 \text{ yr}}\right)\left(\frac{25 \text{ h}}{1 \text{ day}}\right)\left(\frac{60 \text{ min}}{1 \text{ h}}\right) = 6 \times 10^5 \text{ min}$$

Notice how much simpler it is in the expression above to multiply 400×25 than it is to work with the more accurate 365×24. These approximate values for the number of days in a year and the number of hours in a day are close enough for our purposes. Thus, in 70 years there will be $(70 \text{ yr})(6 \times 10^5 \text{ min/yr}) = 4 \times 10^7$ min. At a rate of 10 breaths/min, an individual would take $\boxed{4 \times 10^8 \text{ breaths}}$ in a lifetime, or on the order of 10^9 breaths.

What If? What if the average life span were estimated as 80 years instead of 70? Would this change our final estimate?

Answer We could claim that $(80 \text{ yr})(6 \times 10^5 \text{ min/yr}) = 5 \times 10^7$ min, so that our final estimate should be 5×10^8 breaths. This is still on the order of 10^9 breaths, so an order-of-magnitude estimate would be unchanged. Furthermore, 80 years is 14% larger than 70 years, but we have overestimated the total time interval by using 400 days in a year instead of 365 and 25 hours in a day instead of 24. These two numbers together result in an overestimate of 14%, which cancels the effect of the increased life span!

Example 1.6 It's a Long Way to San Jose

Estimate the number of steps a person would take walking from New York to Los Angeles.

Solution Without looking up the distance between these two cities, you might remember from a geography class that they are about 3 000 mi apart. The next approximation we must make is the length of one step. Of course, this length depends on the person doing the walking, but we can estimate that each step covers about 2 ft. With our estimated step size, we can determine the number of steps in 1 mi. Because this is a rough calculation, we round 5 280 ft/mi to 5 000 ft/mi. (What percentage error does this introduce?) This conversion factor gives us

$$\frac{5\,000 \cancel{\text{ft}}/\text{mi}}{2 \cancel{\text{ft}}/\text{step}} = 2\,500 \text{ steps/mi}$$

Now we switch to scientific notation so that we can do the calculation mentally:

$$(3 \times 10^3 \cancel{\text{mi}})(2.5 \times 10^3 \text{ steps}/\cancel{\text{mi}})$$
$$= \boxed{7.5 \times 10^6 \text{ steps} \sim 10^7 \text{ steps}}$$

So if we intend to walk across the United States, it will take us on the order of ten million steps. This estimate is almost certainly too small because we have not accounted for curving roads and going up and down hills and mountains. Nonetheless, it is probably within an order of magnitude of the correct answer.

Example 1.7 How Much Gas Do We Use?

Estimate the number of gallons of gasoline used each year by all the cars in the United States.

Solution Because there are about 280 million people in the United States, an estimate of the number of cars in the country is 100 million (guessing that there are between two and three people per car). We also estimate that the average distance each car travels per year is 10 000 mi. If we assume a gasoline consumption of 20 mi/gal or 0.05 gal/mi, then each car uses about 500 gal/yr. Multiplying this by the total number of cars in the United States gives an estimated total consumption of $\boxed{5 \times 10^{10} \text{ gal} \sim 10^{11} \text{ gal.}}$

1.7 Significant Figures

When certain quantities are measured, the measured values are known only to within the limits of the experimental uncertainty. The value of this uncertainty can depend on various factors, such as the quality of the apparatus, the skill of the experimenter, and the number of measurements performed. The number of **significant figures** in a measurement can be used to express something about the uncertainty.

As an example of significant figures, suppose that we are asked in a laboratory experiment to measure the area of a computer disk label using a meter stick as a measuring instrument. Let us assume that the accuracy to which we can measure the length of the label is ± 0.1 cm. If the length is measured to be 5.5 cm, we can claim only that its length lies somewhere between 5.4 cm and 5.6 cm. In this case, we say that the measured value has two significant figures. Note that the significant figures include the first estimated digit. Likewise, if the label's width is measured to be 6.4 cm, the actual value lies between 6.3 cm and 6.5 cm. Thus we could write the measured values as (5.5 ± 0.1) cm and (6.4 ± 0.1) cm.

Now suppose we want to find the area of the label by multiplying the two measured values. If we were to claim the area is $(5.5 \text{ cm})(6.4 \text{ cm}) = 35.2 \text{ cm}^2$, our answer would be unjustifiable because it contains three significant figures, which is greater than the number of significant figures in either of the measured quantities. A good rule of thumb to use in determining the number of significant figures that can be claimed in a multiplication or a division is as follows:

When multiplying several quantities, the number of significant figures in the final answer is the same as the number of significant figures in the quantity having the lowest number of significant figures. The same rule applies to division.

Applying this rule to the previous multiplication example, we see that the answer for the area can have only two significant figures because our measured quantities have only two significant figures. Thus, all we can claim is that the area is 35 cm^2, realizing that the value can range between $(5.4 \text{ cm})(6.3 \text{ cm}) = 34 \text{ cm}^2$ and $(5.6 \text{ cm})(6.5 \text{ cm}) = 36 \text{ cm}^2$.

Zeros may or may not be significant figures. Those used to position the decimal point in such numbers as 0.03 and 0.007 5 are not significant. Thus, there are one and two significant figures, respectively, in these two values. When the zeros come after other digits, however, there is the possibility of misinterpretation. For example, suppose the mass of an object is given as 1 500 g. This value is ambiguous because we do not know whether the last two zeros are being used to locate the decimal point or whether they represent significant figures in the measurement. To remove this ambiguity, it is common to use scientific notation to indicate the number of significant figures. In this case, we would express the mass as 1.5×10^3 g if there are two significant figures in the measured value, 1.50×10^3 g if there are three significant figures, and 1.500×10^3 g if there are four. The same rule holds for numbers less than 1, so that 2.3×10^{-4} has two significant figures (and so could be written 0.000 23) and 2.30×10^{-4} has three significant figures (also written 0.000 230). In general, **a significant figure in a measurement is a reliably known digit (other than a zero used to locate the decimal point) or the first estimated digit.**

For addition and subtraction, you must consider the number of decimal places when you are determining how many significant figures to report:

When numbers are added or subtracted, the number of decimal places in the result should equal the smallest number of decimal places of any term in the sum.

PITFALL PREVENTION

1.6 Read Carefully

Notice that the rule for addition and subtraction is different from that for multiplication and division. For addition and subtraction, the important consideration is the number of *decimal places,* not the number of *significant figures.*

For example, if we wish to compute 123 + 5.35, the answer is 128 and not 128.35. If we compute the sum 1.000 1 + 0.000 3 = 1.000 4, the result has five significant figures, even though one of the terms in the sum, 0.000 3, has only one significant figure. Likewise, if we perform the subtraction 1.002 − 0.998 = 0.004, the result has only one significant figure even though one term has four significant figures and the other has three. In this book, **most of the numerical examples and end-of-chapter problems will yield answers having three significant figures.** When carrying out estimates we shall typically work with a single significant figure.

If the number of significant figures in the result of an addition or subtraction must be reduced, there is a general rule for rounding off numbers, which states that the last digit retained is to be increased by 1 if the last digit dropped is greater than 5. If the last digit dropped is less than 5, the last digit retained remains as it is. If the last digit dropped is equal to 5, the remaining digit should be rounded to the nearest even number. (This helps avoid accumulation of errors in long arithmetic processes.)

A technique for avoiding error accumulation is to delay rounding of numbers in a long calculation until you have the final result. Wait until you are ready to copy the final answer from your calculator before rounding to the correct number of significant figures.

Quick Quiz 1.4 Suppose you measure the position of a chair with a meter stick and record that the center of the seat is 1.043 860 564 2 m from a wall. What would a reader conclude from this recorded measurement?

Example 1.8 Installing a Carpet

A carpet is to be installed in a room whose length is measured to be 12.71 m and whose width is measured to be 3.46 m. Find the area of the room.

Solution If you multiply 12.71 m by 3.46 m on your calculator, you will see an answer of 43.976 6 m^2. How many of these numbers should you claim? Our rule of thumb for multiplication tells us that you can claim only the number of significant figures in your answer as are present in the measured quantity having the lowest number of significant figures. In this example, the lowest number of significant figures is three in 3.46 m, so we should express our final answer as 44.0 m^2.

SUMMARY

Take a practice test for this chapter by clicking on the Practice Test link at http://www.pse6.com.

The three fundamental physical quantities of mechanics are length, mass, and time, which in the SI system have the units meters (m), kilograms (kg), and seconds (s), respectively. Prefixes indicating various powers of ten are used with these three basic units.

The **density** of a substance is defined as its *mass per unit volume.* Different substances have different densities mainly because of differences in their atomic masses and atomic arrangements.

The method of **dimensional analysis** is very powerful in solving physics problems. Dimensions can be treated as algebraic quantities. By making estimates and performing order-of-magnitude calculations, you should be able to approximate the answer to a problem when there is not enough information available to completely specify an exact solution.

When you compute a result from several measured numbers, each of which has a certain accuracy, you should give the result with the correct number of **significant figures.** When multiplying several quantities, the number of significant figures in the final answer is the same as the number of significant figures in the quantity having the lowest number of significant figures. The same rule applies to division. When numbers are added or subtracted, the number of decimal places in the result should equal the smallest number of decimal places of any term in the sum.

QUESTIONS

1. What types of natural phenomena could serve as time standards?

$\boxed{\textbf{2.}}$ Suppose that the three fundamental standards of the metric system were length, *density,* and time rather than length, *mass,* and time. The standard of density in this system is to be defined as that of water. What considerations about water would you need to address to make sure that the standard of density is as accurate as possible?

3. The height of a horse is sometimes given in units of "hands." Why is this a poor standard of length?

4. Express the following quantities using the prefixes given in Table 1.4: (a) 3×10^{-4} m (b) 5×10^{-5} s (c) 72×10^{2} g.

$\boxed{\textbf{5.}}$ Suppose that two quantities A and B have different dimensions. Determine which of the following arithmetic operations *could* be physically meaningful: (a) $A + B$ (b) A/B (c) $B - A$ (d) AB.

6. If an equation is dimensionally correct, does this mean that the equation must be true? If an equation is not dimensionally correct, does this mean that the equation cannot be true?

7. Do an order-of-magnitude calculation for an everyday situation you encounter. For example, how far do you walk or drive each day?

8. Find the order of magnitude of your age in seconds.

9. What level of precision is implied in an order-of-magnitude calculation?

10. Estimate the mass of this textbook in kilograms. If a scale is available, check your estimate.

11. In reply to a student's question, a guard in a natural history museum says of the fossils near his station, "When I started work here twenty-four years ago, they were eighty million years old, so you can add it up." What should the student conclude about the age of the fossils?

PROBLEMS

1, 2, 3 = straightforward, intermediate, challenging $\boxed{}$ = full solution available in the *Student Solutions Manual and Study Guide*

= coached solution with hints available at http://www.pse6.com = computer useful in solving problem

= paired numerical and symbolic problems

Section 1.2 Matter and Model Building

Note: Consult the endpapers, appendices, and tables in the text whenever necessary in solving problems. For this chapter, Appendix B.3 may be particularly useful. Answers to odd-numbered problems appear in the back of the book.

1. A crystalline solid consists of atoms stacked up in a repeating lattice structure. Consider a crystal as shown in Figure P1.1a. The atoms reside at the corners of cubes of side $L = 0.200$ nm. One piece of evidence for the regular arrangement of atoms comes from the flat surfaces along which a crystal separates, or cleaves, when it is broken. Suppose this crystal cleaves along a face diagonal, as shown in Figure P1.1b. Calculate the spacing d between two adjacent atomic planes that separate when the crystal cleaves.

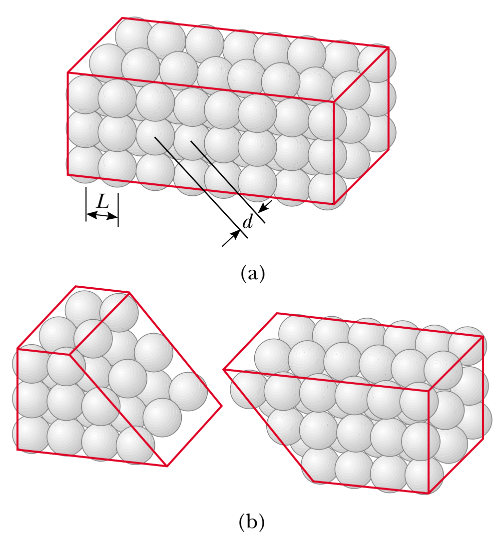

Figure P1.1

Section 1.3 Density and Atomic Mass

2. Use information on the endpapers of this book to calculate the average density of the Earth. Where does the value fit among those listed in Tables 1.5 and 14.1? Look up the density of a typical surface rock like granite in another source and compare the density of the Earth to it.

3. The standard kilogram is a platinum–iridium cylinder 39.0 mm in height and 39.0 mm in diameter. What is the density of the material?

4. A major motor company displays a die-cast model of its first automobile, made from 9.35 kg of iron. To celebrate its hundredth year in business, a worker will recast the model in gold from the original dies. What mass of gold is needed to make the new model?

5. What mass of a material with density ρ is required to make a hollow spherical shell having inner radius r_1 and outer radius r_2?

6. Two spheres are cut from a certain uniform rock. One has radius 4.50 cm. The mass of the other is five times greater. Find its radius.

7. Calculate the mass of an atom of (a) helium, (b) iron, and (c) lead. Give your answers in grams. The atomic masses of these atoms are 4.00 u, 55.9 u, and 207 u, respectively.

8. The paragraph preceding Example 1.1 in the text mentions that the atomic mass of aluminum is $27.0\,\text{u} = 27.0 \times 1.66 \times 10^{-27}$ kg. Example 1.1 says that 27.0 g of aluminum contains 6.02×10^{23} atoms. (a) Prove that each one of these two statements implies the other. (b) **What If?** What if it's not aluminum? Let M represent the numerical value of the mass of one atom of any chemical element in atomic mass units. Prove that M grams of the substance contains a particular number of atoms, the same number for all elements. Calculate this number precisely from the value for u quoted in the text. The number of atoms in M grams of an element is called *Avogadro's number* N_A. The idea can be extended: Avogadro's number of molecules of a chemical compound has a mass of M grams, where M atomic mass units is the mass of one molecule. Avogadro's number of atoms or molecules is called one *mole,* symbolized as 1 mol. A periodic table of the elements, as in Appendix C, and the chemical formula for a compound contain enough information to find the molar mass of the compound. (c) Calculate the mass of one mole of water, H_2O. (d) Find the molar mass of CO_2.

9. On your wedding day your lover gives you a gold ring of mass 3.80 g. Fifty years later its mass is 3.35 g. On the average, how many atoms were abraded from the ring during each second of your marriage? The atomic mass of gold is 197 u.

10. A small cube of iron is observed under a microscope. The edge of the cube is 5.00×10^{-6} cm long. Find (a) the mass of the cube and (b) the number of iron atoms in the cube. The atomic mass of iron is 55.9 u, and its density is 7.86 g/cm^3.

11. A structural I beam is made of steel. A view of its cross-section and its dimensions are shown in Figure P1.11. The density of the steel is 7.56×10^3 kg/m^3. (a) What is the mass of a section 1.50 m long? (b) Assume that the atoms are predominantly iron, with atomic mass 55.9 u. How many atoms are in this section?

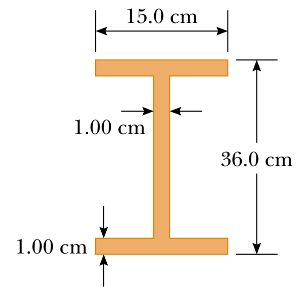

Figure P1.11

12. A child at the beach digs a hole in the sand and uses a pail to fill it with water having a mass of 1.20 kg. The mass of one molecule of water is 18.0 u. (a) Find the number of water molecules in this pail of water. (b) Suppose the quantity of water on Earth is constant at 1.32×10^{21} kg. How many of the water molecules in this pail of water are likely to have been in an equal quantity of water that once filled one particular claw print left by a Tyrannosaur hunting on a similar beach?

Section 1.4 Dimensional Analysis

13. The position of a particle moving under uniform acceleration is some function of time and the acceleration. Suppose we write this position $s = ka^mt^n$, where k is a dimensionless constant. Show by dimensional analysis that this expression is satisfied if $m = 1$ and $n = 2$. Can this analysis give the value of k?

14. Figure P1.14 shows a *frustrum of a cone.* Of the following mensuration (geometrical) expressions, which describes (a) the total circumference of the flat circular faces (b) the volume (c) the area of the curved surface? (i) $\pi(r_1 + r_2)[h^2 + (r_1 - r_2)^2]^{1/2}$ (ii) $2\pi(r_1 + r_2)$ (iii) $\pi h(r_1{}^2 + r_1r_2 + r_2{}^2)$.

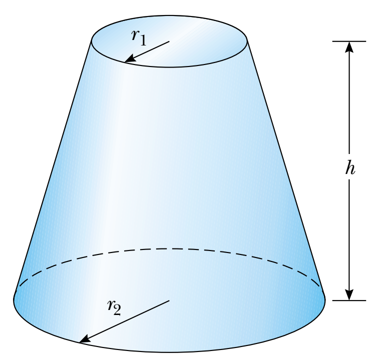

Figure P1.14

15. Which of the following equations are dimensionally correct?
(a) $v_f = v_i + ax$
(b) $y = (2\text{ m})\cos(kx)$, where $k = 2\text{ m}^{-1}$.

16. (a) A fundamental law of motion states that the acceleration of an object is directly proportional to the resultant force exerted on the object and inversely proportional to its mass. If the proportionality constant is defined to have no dimensions, determine the dimensions of force. (b) The newton is the SI unit of force. According to the results for (a), how can you express a force having units of newtons using the fundamental units of mass, length, and time?

17. Newton's law of universal gravitation is represented by

$$F = \frac{GMm}{r^2}$$

Here F is the magnitude of the gravitational force exerted by one small object on another, M and m are the masses of the objects, and r is a distance. Force has the SI units $\text{kg}\cdot\text{m/s}^2$. What are the SI units of the proportionality constant G?

Section 1.5 Conversion of Units

18. A worker is to paint the walls of a square room 8.00 ft high and 12.0 ft along each side. What surface area in square meters must she cover?

19. Suppose your hair grows at the rate 1/32 in. per day. Find the rate at which it grows in nanometers per second. Because the distance between atoms in a molecule is on the order of 0.1 nm, your answer suggests how rapidly layers of atoms are assembled in this protein synthesis.

20. The volume of a wallet is 8.50 in.^{3} Convert this value to m^3, using the definition 1 in. = 2.54 cm.

21. A rectangular building lot is 100 ft by 150 ft. Determine the area of this lot in m^2.

22. An auditorium measures 40.0 m × 20.0 m × 12.0 m. The density of air is 1.20 kg/m^3. What are (a) the volume of the room in cubic feet and (b) the weight of air in the room in pounds?

23. Assume that it takes 7.00 minutes to fill a 30.0-gal gasoline tank. (a) Calculate the rate at which the tank is filled in gallons per second. (b) Calculate the rate at which the tank is filled in cubic meters per second. (c) Determine the time interval, in hours, required to fill a 1-m^3 volume at the same rate. (1 U.S. gal = 231 in.^{3})

24. Find the height or length of these natural wonders in kilometers, meters and centimeters. (a) The longest cave system in the world is the Mammoth Cave system in central Kentucky. It has a mapped length of 348 mi. (b) In the United States, the waterfall with the greatest single drop is Ribbon Falls, which falls 1 612 ft. (c) Mount McKinley in Denali National Park, Alaska, is America's highest mountain at a height of 20 320 ft. (d) The deepest canyon in the United States is King's Canyon in California with a depth of 8 200 ft.

25. A solid piece of lead has a mass of 23.94 g and a volume of 2.10 cm^3. From these data, calculate the density of lead in SI units (kg/m^3).

26. A *section* of land has an area of 1 square mile and contains 640 acres. Determine the number of square meters in 1 acre.

27. An ore loader moves 1 200 tons/h from a mine to the surface. Convert this rate to lb/s, using 1 ton = 2 000 lb.

28. (a) Find a conversion factor to convert from miles per hour to kilometers per hour. (b) In the past, a federal law mandated that highway speed limits would be 55 mi/h. Use the conversion factor of part (a) to find this speed in kilometers per hour. (c) The maximum highway speed is now 65 mi/h in some places. In kilometers per hour, how much increase is this over the 55 mi/h limit?

29. At the time of this book's printing, the U.S. national debt is about \$6 trillion. (a) If payments were made at the rate of \$1 000 per second, how many years would it take to pay off the debt, assuming no interest were charged? (b) A dollar bill is about 15.5 cm long. If six trillion dollar bills were laid end to end around the Earth's equator, how many times would they encircle the planet? Take the radius of the Earth at the equator to be 6 378 km. (*Note:* Before doing any of these calculations, try to guess at the answers. You may be very surprised.)

30. The mass of the Sun is 1.99×10^{30} kg, and the mass of an atom of hydrogen, of which the Sun is mostly composed, is 1.67×10^{-27} kg. How many atoms are in the Sun?

31. One gallon of paint (volume = 3.78×10^{-3} m^3) covers an area of 25.0 m^2. What is the thickness of the paint on the wall?

32. A pyramid has a height of 481 ft and its base covers an area of 13.0 acres (Fig. P1.32). If the volume of a pyramid is given by the expression $V = \frac{1}{3}Bh$, where B is the area of the base and h is the height, find the volume of this pyramid in cubic meters. (1 acre = 43 560 ft^2)

Sylvain Grandadam/Photo Researchers, Inc.

Figure P1.32 Problems 32 and 33.

33. The pyramid described in Problem 32 contains approximately 2 million stone blocks that average 2.50 tons each. Find the weight of this pyramid in pounds.

34. Assuming that 70% of the Earth's surface is covered with water at an average depth of 2.3 mi, estimate the mass of the water on the Earth in kilograms.

35. A hydrogen atom has a diameter of approximately 1.06×10^{-10} m, as defined by the diameter of the spherical electron cloud around the nucleus. The hydrogen nucleus has a diameter of approximately 2.40×10^{-15} m. (a) For a scale model, represent the diameter of the hydrogen atom by the length of an American football field

(100 yd = 300 ft), and determine the diameter of the nucleus in millimeters. (b) The atom is how many times larger in volume than its nucleus?

36. The nearest stars to the Sun are in the Alpha Centauri multiple-star system, about 4.0×10^{13} km away. If the Sun, with a diameter of 1.4×10^9 m, and Alpha Centauri A are both represented by cherry pits 7.0 mm in diameter, how far apart should the pits be placed to represent the Sun and its neighbor to scale?

37. The diameter of our disk-shaped galaxy, the Milky Way, is about 1.0×10^5 lightyears (ly). The distance to Messier 31, which is Andromeda, the spiral galaxy nearest to the Milky Way, is about 2.0 million ly. If a scale model represents the Milky Way and Andromeda galaxies as dinner plates 25 cm in diameter, determine the distance between the two plates.

38. The mean radius of the Earth is 6.37×10^6 m, and that of the Moon is 1.74×10^8 cm. From these data calculate (a) the ratio of the Earth's surface area to that of the Moon and (b) the ratio of the Earth's volume to that of the Moon. Recall that the surface area of a sphere is $4\pi r^2$ and the volume of a sphere is $\frac{4}{3}\pi r^3$.

39. One cubic meter ($1.00\ \text{m}^3$) of aluminum has a mass of 2.70×10^3 kg, and $1.00\ \text{m}^3$ of iron has a mass of 7.86×10^3 kg. Find the radius of a solid aluminum sphere that will balance a solid iron sphere of radius 2.00 cm on an equal-arm balance.

40. Let ρ_{Al} represent the density of aluminum and ρ_{Fe} that of iron. Find the radius of a solid aluminum sphere that balances a solid iron sphere of radius r_{Fe} on an equal-arm balance.

Section 1.6 Estimates and Order-of-Magnitude Calculations

41. Estimate the number of Ping-Pong balls that would fit into a typical-size room (without being crushed). In your solution state the quantities you measure or estimate and the values you take for them.

42. An automobile tire is rated to last for 50 000 miles. To an order of magnitude, through how many revolutions will it turn? In your solution state the quantities you measure or estimate and the values you take for them.

43. Grass grows densely everywhere on a quarter-acre plot of land. What is the order of magnitude of the number of blades of grass on this plot? Explain your reasoning. Note that 1 acre = 43 560 ft^2.

44. Approximately how many raindrops fall on a one-acre lot during a one-inch rainfall? Explain your reasoning.

45. Compute the order of magnitude of the mass of a bathtub half full of water. Compute the order of magnitude of the mass of a bathtub half full of pennies. In your solution list the quantities you take as data and the value you measure or estimate for each.

46. Soft drinks are commonly sold in aluminum containers. To an order of magnitude, how many such containers are thrown away or recycled each year by U.S. consumers? How many tons of aluminum does this represent? In your solution state the quantities you measure or estimate and the values you take for them.

47. To an order of magnitude, how many piano tuners are in New York City? The physicist Enrico Fermi was famous for asking questions like this on oral Ph.D. qualifying examinations. His own facility in making order-of-magnitude calculations is exemplified in Problem 45.48.

Section 1.7 Significant Figures

Note: Appendix B.8 on propagation of uncertainty may be useful in solving some problems in this section.

48. A rectangular plate has a length of (21.3 ± 0.2) cm and a width of (9.8 ± 0.1) cm. Calculate the area of the plate, including its uncertainty.

49. The radius of a circle is measured to be (10.5 ± 0.2) m. Calculate the (a) area and (b) circumference of the circle and give the uncertainty in each value.

50. How many significant figures are in the following numbers? (a) 78.9 ± 0.2 (b) 3.788×10^9 (c) 2.46×10^{-6} (d) 0.005 3.

51. The radius of a solid sphere is measured to be (6.50 ± 0.20) cm, and its mass is measured to be (1.85 ± 0.02) kg. Determine the density of the sphere in kilograms per cubic meter and the uncertainty in the density.

52. Carry out the following arithmetic operations: (a) the sum of the measured values 756, 37.2, 0.83, and 2.5; (b) the product $0.003\,2 \times 356.3$; (c) the product $5.620 \times \pi$.

53. The *tropical year,* the time from vernal equinox to the next vernal equinox, is the basis for our calendar. It contains 365.242 199 days. Find the number of seconds in a tropical year.

54. A farmer measures the distance around a rectangular field. The length of the long sides of the rectangle is found to be 38.44 m, and the length of the short sides is found to be 19.5 m. What is the total distance around the field?

55. A sidewalk is to be constructed around a swimming pool that measures (10.0 ± 0.1) m by (17.0 ± 0.1) m. If the sidewalk is to measure (1.00 ± 0.01) m wide by (9.0 ± 0.1) cm thick, what volume of concrete is needed, and what is the approximate uncertainty of this volume?

Additional Problems

56. In a situation where data are known to three significant digits, we write 6.379 m = 6.38 m and 6.374 m = 6.37 m. When a number ends in 5, we arbitrarily choose to write 6.375 m = 6.38 m. We could equally well write 6.375 m = 6.37 m, "rounding down" instead of "rounding up," because we would change the number 6.375 by equal increments in both cases. Now consider an order-of-magnitude estimate, in which we consider factors rather than increments. We write 500 m $\sim 10^3$ m because 500 differs from 100 by a factor of 5 while it differs from 1 000 by only a factor of 2. We write 437 m $\sim 10^3$ m and 305 m $\sim 10^2$ m. What distance differs from 100 m and from 1 000 m by equal factors, so that we could equally well choose to represent its order of magnitude either as $\sim 10^2$ m or as $\sim 10^3$ m?

57. For many electronic applications, such as in computer chips, it is desirable to make components as small as possible to keep the temperature of the components low and to increase the speed of the device. Thin metallic coatings (films) can be used instead of wires to make electrical connections. Gold is especially useful because it does not oxidize readily. Its atomic mass is 197 u. A gold film can be no thinner than the size of a gold atom. Calculate the minimum coating thickness, assuming that a gold atom occupies a cubical volume in the film that is equal to the volume it occupies in a large piece of metal. This geometric model yields a result of the correct order of magnitude.

58. The basic function of the carburetor of an automobile is to "atomize" the gasoline and mix it with air to promote rapid combustion. As an example, assume that 30.0 cm^3 of gasoline is atomized into N spherical droplets, each with a radius of 2.00×10^{-5} m. What is the total surface area of these N spherical droplets?

59. The consumption of natural gas by a company satisfies the empirical equation $V = 1.50t + 0.008\,00t^2$, where V is the volume in millions of cubic feet and t the time in months. Express this equation in units of cubic feet and seconds. Assign proper units to the coefficients. Assume a month is equal to 30.0 days.

60. In physics it is important to use mathematical approximations. Demonstrate that for small angles ($< 20°$)

$$\tan \alpha \approx \sin \alpha \approx \alpha = \pi\alpha'/180°$$

where α is in radians and α' is in degrees. Use a calculator to find the largest angle for which $\tan \alpha$ may be approximated by $\sin \alpha$ if the error is to be less than 10.0%.

61. A high fountain of water is located at the center of a circular pool as in Figure P1.61. Not wishing to get his feet wet, a student walks around the pool and measures its circumference to be 15.0 m. Next, the student stands at the edge of the pool and uses a protractor to gauge the angle of elevation of the top of the fountain to be 55.0°. How high is the fountain?

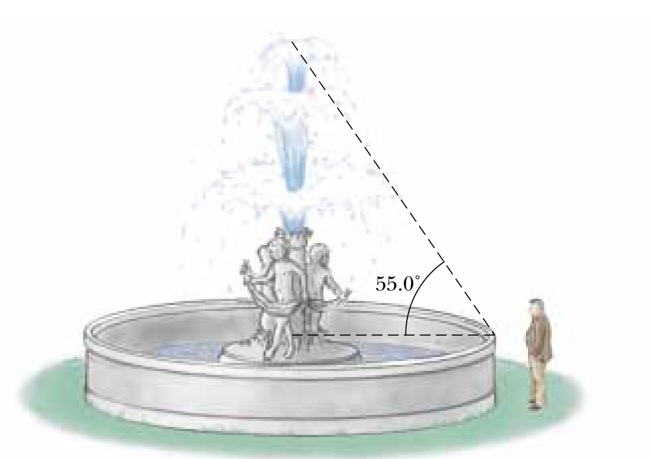

Figure P1.61

62. Collectible coins are sometimes plated with gold to enhance their beauty and value. Consider a commemorative quarter-dollar advertised for sale at \$4.98. It has a diameter of 24.1 mm, a thickness of 1.78 mm, and is completely covered with a layer of pure gold 0.180 μm thick. The volume of the plating is equal to the thickness of the layer times the area to which it is applied. The patterns on the faces of the coin and the grooves on its edge have a negligible effect on its area. Assume that the price of gold is \$10.0 per gram. Find the cost of the gold added to the coin. Does the cost of the gold significantly enhance the value of the coin?

63. There are nearly $\pi \times 10^7$ s in one year. Find the percentage error in this approximation, where "percentage error" is defined as

$$\text{Percentage error} = \frac{|\text{assumed value} - \text{true value}|}{\text{true value}} \times 100\%$$

64. Assume that an object covers an area A and has a uniform height h. If its cross-sectional area is uniform over its height, then its volume is given by $V = Ah$. (a) Show that $V = Ah$ is dimensionally correct. (b) Show that the volumes of a cylinder and of a rectangular box can be written in the form $V = Ah$, identifying A in each case. (Note that A, sometimes called the "footprint" of the object, can have any shape and the height can be replaced by average thickness in general.)

65. A child loves to watch as you fill a transparent plastic bottle with shampoo. Every horizontal cross-section is a circle, but the diameters of the circles have different values, so that the bottle is much wider in some places than others. You pour in bright green shampoo with constant volume flow rate 16.5 cm^3/s. At what rate is its level in the bottle rising (a) at a point where the diameter of the bottle is 6.30 cm and (b) at a point where the diameter is 1.35 cm?

66. One cubic centimeter of water has a mass of 1.00×10^{-3} kg. (a) Determine the mass of 1.00 m^3 of water. (b) Biological substances are 98% water. Assume that they have the same density as water to estimate the masses of a cell that has a diameter of 1.0 μm, a human kidney, and a fly. Model the kidney as a sphere with a radius of 4.0 cm and the fly as a cylinder 4.0 mm long and 2.0 mm in diameter.

67. Assume there are 100 million passenger cars in the United States and that the average fuel consumption is 20 mi/gal of gasoline. If the average distance traveled by each car is 10 000 mi/yr, how much gasoline would be saved per year if average fuel consumption could be increased to 25 mi/gal?

68. A creature moves at a speed of 5.00 furlongs per fortnight (not a very common unit of speed). Given that 1 furlong = 220 yards and 1 fortnight = 14 days, determine the speed of the creature in m/s. What kind of creature do you think it might be?

69. The distance from the Sun to the nearest star is about 4×10^{16} m. The Milky Way galaxy is roughly a disk of diameter $\sim 10^{21}$ m and thickness $\sim 10^{19}$ m. Find the order of magnitude of the number of stars in the Milky Way. Assume the distance between the Sun and our nearest neighbor is typical.

70. The data in the following table represent measurements of the masses and dimensions of solid cylinders of aluminum, copper, brass, tin, and iron. Use these data to calculate the densities of these substances. Compare your results for aluminum, copper, and iron with those given in Table 1.5.

Substance	Mass (g)	Diameter (cm)	Length (cm)
Aluminum	51.5	2.52	3.75
Copper	56.3	1.23	5.06
Brass	94.4	1.54	5.69
Tin	69.1	1.75	3.74
Iron	216.1	1.89	9.77

71. (a) How many seconds are in a year? (b) If one micrometeorite (a sphere with a diameter of 1.00×10^{-6} m) strikes each square meter of the Moon each second, how many years will it take to cover the Moon to a depth of 1.00 m? To solve this problem, you can consider a cubic box on the Moon 1.00 m on each edge, and find how long it will take to fill the box.

Answers to Quick Quizzes

1.1 (a). Because the density of aluminum is smaller than that of iron, a larger volume of aluminum is required for a given mass than iron.

1.2 False. Dimensional analysis gives the units of the proportionality constant but provides no information about its numerical value. To determine its numerical value requires either experimental data or geometrical reasoning. For example, in the generation of the equation $x = \frac{1}{2}at^2$, because the factor $\frac{1}{2}$ is dimensionless, there is no way of determining it using dimensional analysis.

1.3 (b). Because kilometers are shorter than miles, a larger number of kilometers is required for a given distance than miles.

1.4 Reporting all these digits implies you have determined the location of the center of the chair's seat to the nearest $\pm 0.000\ 000\ 000\ 1$ m. This roughly corresponds to being able to count the atoms in your meter stick because each of them is about that size! It would be better to record the measurement as 1.044 m: this indicates that you know the position to the nearest millimeter, assuming the meter stick has millimeter markings on its scale.